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Cosmological reionization

Piero Madau

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Cosmological reionization **llogical reioniza
By PIERO MADAU**

IDERO MADAU
Institute of Astronomy, University of Cambridge,
*Madinaley Boad, Cambridge CB*⁸ 0HA *UK Madingley Road, Cambridge CB3 0HA, UK*

 $Madingley Road, Cambridge CB3 OHA, UK$
In popular cosmological scenarios, some time beyond a redshift of 10, stars within protogalaxies created the first heavy elements; these systems, together perhaps with In popular cosmological scenarios, some time beyond a redshift of 10, stars within
protogalaxies created the first heavy elements; these systems, together perhaps with
an early population of quasars, generated the UV radia protogalaxies created the first heavy elements; these systems, together perhaps with
an early population of quasars, generated the UV radiation and mechanical energy
that reheated and reionized the cosmos. The history of t an early population of quasars, generated the UV radiation and mechanical energy
that reheated and reionized the cosmos. The history of the Universe during and
soon after these crucial formative stages is recorded in the a that reheated and reionized the cosmos. The history of the Universe during and
soon after these crucial formative stages is recorded in the all-pervading intergalactic
medium (IGM), which contains most of the ordinary bary soon after these crucial formative stages is recorded in the all-pervading intergalactic
medium (IGM), which contains most of the ordinary baryonic material left over from
the big bang. Throughout the epoch of structure fo medium (IGM), which contains most of the ordinary baryonic material left over from
the big bang. Throughout the epoch of structure formation, the IGM becomes clumpy
and acquires peculiar motions under the influence of grav the big bang. Throughout the epoch of structure formation, the IGM becomes clumpy
and acquires peculiar motions under the influence of gravity, and acts as a source for
the gas that gets accreted, cools and forms stars wit and acquires peculiar motions under the influence of gravity, and act
the gas that gets accreted, cools and forms stars within galaxies, a
the metal-enriched material, energy and radiation that they eject. the metal-enriched material, energy and radiation that they eject.
Keywords: diffuse radiation; intergalactic medium; radiative transfer

1. Introduction

1. **Introduction**
At epochs corresponding to $z \sim 1000$, the intergalactic medium (IGM) is expected
to recombine and remain neutral until sources of radiation and heat develop that At epochs corresponding to $z \sim 1000$, the intergalactic medium (IGM) is expected
to recombine and remain neutral until sources of radiation and heat develop that
are capable of reionizing it. The detection of transmitted At epochs corresponding to $z \sim 1000$, the intergalactic medium (IGM) is expected
to recombine and remain neutral until sources of radiation and heat develop that
are capable of reionizing it. The detection of transmitted to recombine and remain neutral until sources of radiation and heat develop that
are capable of reionizing it. The detection of transmitted flux shortward of the Ly α
wavelength in the spectra of sources at $z \sim 5$ impl are capable of reionizing it. The detection of transmitted flux shortward of the $Ly\alpha$ wavelength in the spectra of sources at $z \sim 5$ implies that the hydrogen component of this IGM was ionized at even higher redshifts. wavelength in the spectra of sources at $z \sim 5$ implies that the hydrogen component
of this IGM was ionized at even higher redshifts. There is some evidence that the
double reionization of helium may have occurred later, double reionization of helium may have occurred later, but this is still controversial. It appears then that substantial sources of UV photons and mechanical energy sial. It appears then that substantial sources of UV photons and mechanical energy
were already present when the Universe was less than 7% of its current age, perhaps
quasars and/or young star-forming galaxies: an episode were already present when the Universe was less than 7% of its current age, perhaps
quasars and/or young star-forming galaxies: an episode of pre-galactic star formation
may provide a possible explanation for the widesprea quasars and/or young star-forming galaxies: an episode of pre-galactic star formation
may provide a possible explanation for the widespread existence of heavy elements
(like carbon, oxygen and silicon) in the IGM, while th (like carbon, oxygen and silicon) in the IGM, while the integrated radiation emitted (like carbon, oxygen and silicon) in the IGM, while the integrated radiation emitted
from quasars is probably responsible for the reionization of the intergalactic helium.
Establishing the epoch of reionization and reheati from quasars is probably responsible for the reionization of the intergalactic helium.
Establishing the epoch of reionization and reheating is crucial for determining its
impact on several key cosmological issues, from the Establishing the epoch of reionization and reheating is crucial for determining its
impact on several key cosmological issues, from the role reionization plays in allowing
protogalactic objects to cool and make stars, to d impact on several key cosmological issues, from the role reionization plays in allowing
protogalactic objects to cool and make stars, to determining the small-scale struc-
ture in the temperature fluctuations of the cosmic protogalactic objects to cool and make stars, to determining the small-scale structure in the temperature fluctuations of the cosmic microwave background (CMB).
Conversely, probing the reionization epoch may provide a mean ture in the temperature fluctuations of the cosmic microwave background (CMB).
Conversely, probing the reionization epoch may provide a means for constraining
competing models for the formation of cosmic structures, and of Conversely, probing the reionization epoch may provide a means for co
competing models for the formation of cosmic structures, and of detecting
of the first generation of stars, galaxies and black holes in the Universe. of the first generation of stars, galaxies and black holes in the Universe.
2. The transition from a neutral to an ionized Universe

2. The transition from a neutral to an ionized Universe
Popular cosmological models predict that most of the intergalactic hydrogen was
reionized by the first generation of stars or accreting black holes at $z = 7-15$. One Popular cosmological models predict that most of the intergalactic hydrogen was reionized by the first generation of stars or accreting black holes at $z = 7{\text -}15$. One reionized by the first generation of stars or accreting black holes at $z = 7{\text -}15$. One
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should note, however, that while numerical N -body + hydrodynamical simulations should note, however, that while numerical N -body + hydrodynamical simulations have convincingly shown that the IGM is expected to fragment into structures at early times in cold dark matter (CDM) cosmogonies (see, for exteed to fragment into structures at the convincingly shown that the IGM is expected to fragment into structures at early times in cold dark matter (CDM) cosmogonies (see, for example, Cen *et al.* 1994; Zhang *et al.* 19 early times in cold dark matter (CDM) cosmogonies (see, for example, Cen *et al.* 1994; Zhang *et al.* 1995; Hernquist *et al.* 1996), the same simulations are much less able to predict the efficiency with which the first up the Universe at the end of the `dark age' (Rees, this issue).

(*a*) *Photoionization versus collisional ionization*

The scenario that has been the subject of the most theoretical study is one in
which hydrogen is photoionized by the UV radiation emitted either by quasars or by
stars with masses $\geq 10 M_{\odot}$, rather than ionized by co The scenario that has been the subject of the most theoretical study is one in The scenario that has been the subject of the most theoretical study is one in which hydrogen is photoionized by the UV radiation emitted either by quasars or by stars with masses $\geq 10M_{\odot}$, rather than ionized by co which hydrogen is photoionized by the UV radiation emitted either by quasars or by
stars with masses $\geq 10 M_{\odot}$, rather than ionized by collisions with electrons heated up
by, for example, supernova-driven winds from stars with masses $\gtrsim 10 M_{\odot}$, rather than ionized by collisions with electrons heated up
by, for example, supernova-driven winds from early pregalactic ('Pop III') objects.
In the former case, a high degree of ioniza by, for example, supernova-driven winds from early pregalactic (Pop III) objects.
In the former case, a high degree of ionization requires *ca*. 13.6(1 + t/\bar{t}_{rec}) eV per
hydrogen atom, where \bar{t}_{rec} is the volu In the former case, a high degree of ionization requires ca. 13.6(1 + t/\bar{t}_{rec}) eV per
hydrogen atom, where \bar{t}_{rec} is the volume-averaged hydrogen recombination time-
scale, t/\bar{t}_{rec} being much greater tha hydrogen atom, where \bar{t}_{rec} is the volume-averaged hydrogen recombination time-
scale, t/\bar{t}_{rec} being much greater than unity already at $z \approx 10$ according to the
numerical simulations of Gnedin & Ostriker (199 scale, t/\bar{t}_{rec} being much greater than unity already at $z \approx 10$ according to the numerical simulations of Gnedin & Ostriker (1997) and Gnedin (2000). Collisional ionization to a neutral fraction of only a few parts numerical simulations of Gnedin & Ostriker (1997) and Gnedin (2000). Colli
ionization to a neutral fraction of only a few parts in 10^5 requires a comparenergy input, i.e. an IGM temperature close to 10^5 K or *ca*. 2 nization to a neutral fraction of only a few parts in 10^5 requires a comparable
ergy input, i.e. an IGM temperature close to 10^5 K or ca 25 eV per atom.
Massive stars will deposit both radiative and mechanical ener $\overline{\sigma}$

energy input, i.e. an IGM temperature close to 10^5 K or ca . 25 eV per atom.
Massive stars will deposit both radiative and mechanical energy into the interstel-
lar medium of Pop III objects. A complex network of 'feed Massive stars will deposit both radiative and mechanical energy into the interstel-
lar medium of Pop III objects. A complex network of 'feedback' mechanisms is likely
to be at work in these systems, as the gas in shallow lar medium of Pop III objects. A complex network of 'feedback' mechanisms is likely
to be at work in these systems, as the gas in shallow potential is more easily blown
away, thereby quenching further star formation (Mac L to be at work in these systems, as the gas in shallow potential is more easily blown
away, thereby quenching further star formation (Mac Low & Ferrara 1999), and the
blast waves produced by supernova explosions reheat the away, thereby quenching further star formation (Mac Low & Ferrara 1999), and the blast waves produced by supernova explosions reheat the surrounding intergalactic gas and enrich it with newly formed heavy elements and dust blast waves produced by supernova explosions reheat the surrounding intergalactic
gas and enrich it with newly formed heavy elements and dust. Therefore, it is dif-
ficult to establish whether an early input of mechanical gas and enrich it with newly formed heavy elements and dust. Therefore, it is difficult to establish whether an early input of mechanical energy will actually play a major role in determining the thermal and ionization sta ficult to establish whether an early input of mechanical energy will actually play a
major role in determining the thermal and ionization state of the IGM on large scales
(Tegmark *et al.* 1993). What can easily be shown i major role in determining the thermal and ionization state of the IGM on large scales (Tegmark *et al.* 1993). What can easily be shown is that, during the evolution of a 'typical' stellar population, more energy is lost (Tegmark *et al.* 1993). What can easily be shown is that, during the evolution of a "typical" stellar population, more energy is lost in UV radiation than in mechanical form. This is because in nuclear burning from zero **MATHEMATICAL,
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& ENGINEERING
SCIENCES** 'typical' stellar population, more energy is lost in UV rad
form. This is because in nuclear burning from zero to sola
the energy radiated per baryon is $0.02 \times 0.007 \times m_{\text{H}}c^2$;
into H-ionizing photons. The same massiv adiation than in mechanical
plar metallicity ($Z_{\odot} = 0.02$),
; about one-third of it goes
dominate the UV light also form. This is because in nuclear burning from zero to solar metallicity ($Z_{\odot} = 0.02$), the energy radiated per baryon is $0.02 \times 0.007 \times m_{\rm H}c^2$; about one-third of it goes into H-ionizing photons. The same massive s $2 \cdot a$ into H-ionizing photons. The same massive stars that dominate the UV light also explode as supernovae (SNe), returning most of the metals to the interstellar medium explode as supernovae (SNe), returning most of the metals to the interstellar medium
and injecting about 10^{51} erg per event in kinetic energy. For a Salpeter initial mass
function (IMF), one has about one supernova (S and injecting about 10^{51} erg per event in kinetic energy. For a Salpeter initial mass function (IMF), one has about one supernova (SN) for every $150M_{\odot}$ of baryons that and injecting about 10^{51} erg per event in kinetic energy. For a Salpeter initial mass
function (IMF), one has about one supernova (SN) for every $150M_{\odot}$ of baryons that
form stars. The mass fraction in mechanical function (IMF), one has about one supernova (SN) for ever
form stars. The mass fraction in mechanical energy is then
lower than the fraction released in photons above 1 ryd.
The relative importance of photoionization versu The relative importance of photoionization versus shock ionization will depend,
we relative importance of photoionization versus shock ionization will depend,
we relative importance of photoionization versus shock ionizat

however than the fraction released in photons above 1 ryd.
The relative importance of photoionization versus shock ionization will depend,
however, on the efficiency with which radiation and mechanical energy actually
esca The relative importance of photoionization versus shock ionization will depend,
however, on the efficiency with which radiation and mechanical energy actually
escape into the IGM. Consider, for example, the case of an ear however, on the efficiency with which radiation and mechanical energy actually
escape into the IGM. Consider, for example, the case of an early generation of halos
with circular speed $v_c = 50 \text{ km s}^{-1}$, corresponding, in escape into the IGM. Consider, for exa
with circular speed $v_c = 50 \text{ km s}^{-1}$, consider the virial temperature $T_v = 0.5 \mu m_p v_c^2 / k$
 10^{9} $(1+z)/(10^{-3}/2h^{-1}M_c + \ln \theta)$ $2/k$ example, the case of an early generation of
, corresponding, in top-hat spherical collap
 $c^2/k \approx 10^{5.3}$ K and halo mass $M = 0.1v_c^3/G$ $_\mathrm{c}^\mathrm{3}/GH\approx$ $10^{9}[(1$ h circular speed $v_c = 50 \text{ km s}^{-1}$, corresponding, in top-hat spherical collapse to
irial temperature $T_v = 0.5 \mu m_p v_c^2 / k \approx 10^{5.3} \text{ K}$ and halo mass $M = 0.1 v_c^3 / GH \approx$
 $[(1 + z)/10]^{-3/2} h^{-1} M_{\odot}$. In these systems, rapid co a virial temperature $T_v = 0.5 \mu m_p v_c^2 / k \approx 10^{5.5}$ K and halo mass $M = 0.1 v_c^3 / GH \approx 10^9 [(1+z)/10]^{-3/2} h^{-1} M_{\odot}$. In these systems, rapid cooling by atomic hydrogen can take place, and a significant fraction, $f \Omega_B$, of the $10^9[(1+z)/10]^{-3/2}h^{-1}M_{\odot}$.[†] In these systems, rapid cooling by atomic hydrogen can take place, and a significant fraction, $f\Omega_{\rm B}$, of their total mass may be converted into stars over a dynamical time-scale. For take place, and a significant fraction, $f\Omega_B$, of their total mass may be converted
into stars over a dynamical time-scale. For $f = 0.05$, $\Omega_B h^2 = 0.02$, and $h = 0.5$,
the explosive output of 50 000 SNe would inject an into stars over a dynamical time-scale. For $f = 0.05$, $\Omega_B h^2 = 0.02$, and $h = 0.5$, the explosive output of 50 000 SNe would inject an energy $E_0 \approx 10^{55.7}$ erg. The hot gas will escape its host, shock the IGM, and eve gas will escape its host, shock the IGM, and eventually form a cosmolog \dagger This assumes an Einstein-de Sitter (EdS) Universe with $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$.
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wave. If the explosion occurs at cosmic time $t = 4 \times 10^8$ yr, corresponding, in the wave. If the explosion occurs at cosmic time $t = 4 \times 10^8$ yr, corresponding, in the adopted cosmology (EdS with $h = 0.5$), to $z = 9$, at time $\Delta t = 0.2t$ after the event, the proper radius of the (adiabatic) shock is giv wave. If the explosion occurs at cosmic time $t = 4 \times 10^8$ yr, corresponding, in the adopted cosmology (EdS with $h = 0.5$), to $z = 9$, at time $\Delta t = 0.2t$ after the event, the proper radius of the (adiabatic) shock is giv the proper radius of the (adiabatic) shock is given by the standard Sedov-Taylor self-similar solution:

self-similar solution:
\n
$$
R_{\rm s} \approx \left(\frac{12\pi G E_0}{\Omega_{\rm B}}\right)^{1/5} t^{2/5} \Delta t^{2/5} \approx 23 \text{ kpc.}
$$
\n(2.1)
\nAt this instant, the shock velocity relative to the Hubble flow is

$$
v_{\rm s} \approx 2R_{\rm s}/5\Delta t \approx 110 \text{ km s}^{-1},\tag{2.2}
$$

 $v_s \approx 2R_s/5\Delta t \approx 110 \text{ km s}^{-1}$, (2.2)
still much higher than the escape velocity from the halo. The gas temperature just
behind the shock front is $T_c = 3\mu m_r v^2/16k \approx 4 \times 10^5 \text{ K}$, more than enough to effi $v_s \sim 2R_s / 3\Delta t \sim 110 \text{ km s}$ (2.2)
still much higher than the escape velocity from the halo. The gas temperature just
behind the shock front is $T_s = 3\mu m_p v_s^2/16k \approx 4 \times 10^5 \text{ K}$, more than enough to effi-
ciently ionize behind the shock front is $T_s = 3\mu m_p v_s^2/16k \approx 4 \times 10^5$ K, more than enough to effistill much higher than the escape velocity from the halo. The gas temperature just
behind the shock front is $T_s = 3\mu m_p v_s^2/16k \approx 4 \times 10^5$ K, more than enough to effi-
ciently ionize all the incoming hydrogen. At these r behind the shock front is $T_s = 3\mu m_p v_s^2/16k \approx 4 \times 10^5$ K, more than enough to efficiently ionize all the incoming hydrogen. At these redshifts, it is the onset of Compton cooling off CMB photons that ends the adiabatic ciently ionize all the incoming hydrogen. At these redshifts, it is the onset of Compton
cooling off CMB photons that ends the adiabatic stage of blast-wave propagation.
According to the Press-Schechter formalism, the com cooling off CMB photons that ends the adiabatic stage of blast-wave p
According to the Press-Schechter formalism, the comoving abundance
dark halos with mass $M = 10^9 h^{-1} M_{\odot}$ at $z = 9$ is $dn/d \ln M \sim 5h^3$ M₁
sponding t 3 Mpc^{-3} , c equation.
|-
| corre-
|-1 kpc. According to the Press–Schechter formalism, the comoving abundance of collapsed
dark halos with mass $M = 10^9 h^{-1} M_{\odot}$ at $z = 9$ is $dn/d \ln M \sim 5h^3$ Mpc⁻³, corre-
sponding to a mean proper distance between neighbouring dark halos with mass $M = 10^9 h^{-1} M_{\odot}$ at $z = 9$ is $dn/d \ln M \sim 5h^3$ Mpc⁻³, corresponding to a mean proper distance between neighbouring halos of *ca*. $40h^{-1}$ kpc, and to a total mass density parameter of the order of sponding to a mean proper distance between neighbouring halos of $ca. 40h^{-1}$ kpc,
and to a total mass density parameter of the order of 0.02. With the assumed star-
formation efficiency, only a small fraction $(ca. 1\%)$ of and to a total mass density parameter of the order of 0.02. With the assumed star-
formation efficiency, only a small fraction $(ca.1\%)$ of the stars seen today would have
to be formed at these early epochs. Still, our sim to be formed at these early epochs. Still, our simple analysis shows that the blast to be formed at these early epochs. Still, our simple analysis shows that the blast
waves from such a population of pregalactic objects could overlap with large enough
velocities to initially drive the IGM to a significan waves from such a population of pregalactic objects could overlap with large enough
velocities to initially drive the IGM to a significantly higher adiabat, $T \geq 10^5$ K,
than expected from photoionization, and pollute t velocities to initially drive the IGM to a significantly higher adiabat, $T \gtrsim 10^5$ K, than expected from photoionization, and pollute the entire IGM with metal-enriched material. A lower density of sources—which would, than expected from photoionization, and pollute the entire IGM with metal-enriched material. A lower density of sources—which would, therefore, have to originate from higher-amplitude peaks—would suffice if the typical eff material. A lower density of sources—which would, therefore, have to originate from gher-amplitude peaks—would suffice if the typical efficiency of star formation were
ger than assumed here.
Quasar-driven blast waves $(E_0 \ge 10^{60} \text{ erg})$ are, instead, quite inefficient at ionizing
e IGM since much of the

larger than assumed here.
Quasar-driven blast waves $(E_0 \gtrsim 10^{60} \text{ erg})$ are, instead, quite inefficient at ionizing
the IGM, since much of the initial explosion energy is lost into the collisionless
component (Voit 1996 Quasar-driven blast waves $(E_0 \gtrsim 10^{60} \text{ erg})$ are, instead, quite inefficient at ionizing
the IGM, since much of the initial explosion energy is lost into the collisionless
component (Voit 1996). They would also be too ATHEMATICAL,
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Ciences the IGM, since much of the initial explosion energy is lost into the collisionless component (Voit 1996). They would also be too rare to fill the IGM without violating the COBE limit on the y -distortion of the microwave

(*b*) *Cosmological HII regions*

In the following sections we will focus our attention on the photoionization of In the following sections we will focus our attention on the photoionization of
the IGM, i.e. we will assume that UV photons from an early generation of stars
and/or quasars are the main source of energy for the reionizat In the following sections we will focus our attention on the photoionization of the IGM, i.e. we will assume that UV photons from an early generation of stars and/or quasars are the main source of energy for the reionizati the IGM, i.e. we will assume that UV photons from an early generation of stars and/or quasars are the main source of energy for the reionization and reheating of the Universe, and that star formation and quasar activity oc and/or quasars are the main source of energy for the reionization and reheating of the Universe, and that star formation and quasar activity occur in collapsed galaxy halos.
The process then begins as individual sources s Universe, and that star formation and quasar activity occur in collapsed galaxy halos.
The process then begins as individual sources start to generate expanding HII regions
in the surrounding IGM; throughout an HII region, The process then begins as individual sources start to generate expanding HII regions
in the surrounding IGM; throughout an HII region, H is ionized and He is either singly
or doubly ionized. As more and more sources of UV in the surrounding IGM; throughout an HII region, H is ionized and He is either singly
or doubly ionized. As more and more sources of UV radiation switch on, the ionized
volume grows in size, while the neutral phase shrink \cong \Box or doubly ionized. As more and more sources of UV radiation switch on, the ionized \Box \Box volume grows in size, while the neutral phase shrinks. Reionization is completed \Box \Box when the HII regions overl volume grows in size, while the neutral phase shrinks. Reionization is con
when the HII regions overlap, and every point in the intergalactic space gets e
for the first time to a nearly uniform Lyman-continuum (Lyc) backgr nen the HII regions overlap, and every point in the intergalactic space gets exposed
the first time to a nearly uniform Lyman-continuum (Lyc) background.
When an isolated point source of ionizing radiation turns on, an ion

When an isolated point source of ionizing radiation turns on, an ionization front separating the HII and HI regions propagates into the neutral gas, and the proper volume V_I of the ionized zone grows according to the equation

$$
\frac{\mathrm{d}V_{\rm I}}{\mathrm{d}t} - 3HV_{\rm I} = \frac{\dot{N}_{\rm ion}}{\bar{n}_{\rm H}} - \frac{V_{\rm I}}{\bar{t}_{\rm rec}},\tag{2.3}
$$

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Figure 1. Simulating the reionization of the Universe: propagation of an ionization front in a $128³$ cosmological density field. A 'mini-quasar' emitting $5 \times 10⁵³$ ionizing photons per second Figure 1. Simulating the reionization of the Universe: propagation of an ionization front in a
128³ cosmological density field. A 'mini-quasar' emitting 5×10^{53} ionizing photons per second
was turned on at the dens 128° cosmological density field. A 'mini-quasar' emitting 5×10^{33} ionizing photons per second
was turned on at the densest cell, in a virialized halo of total mass $10^{11} M_{\odot}$. The box length
is 2.4 comoving Mpc. is 2.4 comoving Mpc. The solid contours give the position of the front at 0.15, 0.25, 0.38 and 0.57 Myr after the quasar has switched on at $z = 7$. The underlying greyscale image indicates the initial HI density field (from Abel *et al.* (1999)).

the initial HI density field (from Abel *et al.* (1999)).
(Shapiro & Giroux 1987), where N_{ion} is the number of ionizing photons emitted
by the central source per unit time $\bar{n}_{\text{U}}(0) = 1.7 \times 10^{-7} (Q_{\text{D}}h^2/0.02) \$ (Shapiro & Giroux 1987), where \dot{N}_{ion} is the number of ionizing photons emitted
by the central source per unit time, $\bar{n}_{\text{H}}(0) = 1.7 \times 10^{-7}$ $(\Omega_{\text{B}}h^2/0.02) \text{ cm}^{-3}$ is the
current mean hydrogen density and a $\left(\Omega_{\rm B}h^2/0\right)$ (Shapiro & Giroux 1987), where N_{ion} is the number of ionizing photons emitted
by the central source per unit time, $\bar{n}_{\text{H}}(0) = 1.7 \times 10^{-7}$ $(\Omega_{\text{B}}h^2/0.02) \text{ cm}^{-3}$ is the
current mean hydrogen density, and all by the central source per unit time, $\bar{n}_{\text{H}}(0) = 1.7 \times 10^{-7}$ ($\Omega_{\text{B}}h^2/0.02$) cm⁻³ is the current mean hydrogen density, and all other symbols have their usual meaning. Most photons travel freely in the ionized current mean hydrogen density, and all other symbols have their usual meaning. Most
photons travel freely in the ionized gas, and are absorbed in a transition layer. In the
case of stellar sources, the ionization front is photons travel freely in the ionized gas, and are absorbed in a transition layer. In the case of stellar sources, the ionization front is quite sharp, and the degree of ionization changes on a short distance of the order case of stellar sources, the ionization front is quite sharp, and the degree of ionization changes on a short distance of the order of the mean free path for an ionizing photon.
When $\bar{t}_{\text{rec}} \ll t$, the growth of the HII changes on a short distance of the order of the mean free path for an ionizing photon.
When $\bar{t}_{\text{rec}} \ll t$, the growth of the HII region is slowed down by recombinations in
the highly inhomogeneous IGM, and its evolution When $\bar{t}_{\text{rec}} \ll t$, the growth of the HII region is slowed down by recombinations in the highly inhomogeneous IGM, and its evolution can be decoupled from the Hubble expansion. Just like in the static case, the ionized the highly inhomogeneous IGM, and its evolution can be dexpansion. Just like in the static case, the ionized bubble
Strömgren sphere after a few recombination time-scales:

$$
V_{\rm I} = \frac{\dot{N}_{\rm ion}\bar{t}_{\rm rec}}{\bar{n}_{\rm H}} (1 - e^{-t/\bar{t}_{\rm rec}}). \tag{2.4}
$$

While the volume that is ionized depends on the luminosity of the central source, While the volume that is ionized depends on the luminosity of the central source,
the time it takes to produce an ionization-bounded region is only a function of \bar{t}_{rec} .
In the presence of a population of ionizing s

hile the volume that is ionized depends on the luminosity of the central source,
e time it takes to produce an ionization-bounded region is only a function of \bar{t}_{rec} .
In the presence of a population of ionizing sour the time it takes to produce an ionization-bounded region is only a function of \bar{t}_{rec} .
In the presence of a population of ionizing sources, the transition from a neutral IGM to one that is almost fully ionized can IGM to one that is almost fully ionized can be statistically described by the evolution with redshift of the *volume-filling factor* (or porosity) Q of HII, HeII, and HeIII regions. The radiation emitted by spatially clust IGM to one that is almost fully ionized can be statistically described by the evo-
lution with redshift of the *volume-filling factor* (or porosity) Q of HII, HeII, and
HeIII regions. The radiation emitted by spatially lution with redshift of the *volume-filling factor* (or porosity) Q of HII, HeII, and HeIII regions. The radiation emitted by spatially clustered stellar-like and quasar-like sources—the number densities and luminositie HeIII regions. The radiation emitted by spatially clustered stellar-like and quasar-like sources—the number densities and luminosities of which may change rapidly as a function of redshift—coupled with absorption processes

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Cosmological reionization 2025
more and more clumpy owing to the nonlinear collapse of structures (figure 1), all
determine the complex topology of neutral and ionized zones in the Universe (Gnedin more and more clumpy owing to the nonlinear collapse of structures (figure 1), all determine the complex topology of neutral and ionized zones in the Universe (Gnedin 2000: Ciardi *et al.* 2000: Abel *et al.* 1999). When more and more clumpy owing to the nonlinear collapse of structures (figure 1), all determine the complex topology of neutral and ionized zones in the Universe (Gnedin 2000; Ciardi *et al.* 2000; Abel *et al.* 1999). When determine the complex topology of neutral and ionized zones in the Universe (Gnedin 2000; Ciardi *et al.* 2000; Abel *et al.* 1999). When $Q \ll 1$ and the radiation sources are randomly distributed, the ionized regions are 2000; Ciardi *et al.* 2000; Abel *et al.* 1999). When $Q \ll 1$ and the radiation sources are randomly distributed, the ionized regions are spatially isolated, every UV photon is absorbed somewhere in the IGM, and the UV ra are randomly distributed, the ionized regions are spatially isolated, every UV photon
is absorbed somewhere in the IGM, and the UV radiation field is highly inhomoge-
neous. As Q grows, the crossing of ionization fronts is absorbed somewhere in the IGM,
neous. As Q grows, the crossing of io
until percolation occurs at $Q = 1$.
Since the mean free path of Lyc ra ous. As Q grows, the crossing of ionization fronts becomes more and more common,
til percolation occurs at $Q = 1$.
Since the mean free path of Lyc radiation is always much smaller than the horizon
nis is also true after

until percolation occurs at $Q = 1$.
Since the mean free path of Lyc radiation is always much smaller than the horizon
(this is also true after 'overlapping' because of the residual HI still present in the
Lyc forest cloud Since the mean free path of Lyc radiation is always much smaller than the horizon (this is also true after 'overlapping' because of the residual HI still present in the Ly α forest clouds and the Lyman-limit systems), t Ly α forest clouds and the Lyman-limit systems), the filling factor of cosmological HII regions is equal at any given time t to the total number of ionizing photons emitted per hydrogen atom by all radiation sources present at earlier epochs,

$$
\int_0^t \dot{n}_{\rm ion} \, {\rm d}t'/\bar{n}_{\rm H},
$$

minus the total number of radiative recombinations per atom,

 \mathbf{r}^t 0 $Q\,{\mathrm d} t'/t_{\mathrm n}$

 $\int_0^{\infty} Q dt'/\bar{t}_{\text{rec}}$.
This statement reflects the simple fact that *every UV photon that is emitted is either*
absorbed by a newly ionized by a pergenting one Differentiating This statement reflects the simple fact that *every UV photon that is emitted is either*
absorbed by a newly ionized hydrogen atom or by a recombining one. Differentiating,
one gets This staten
absorbed by
one gets

$$
\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\dot{n}_{\text{ion}}}{\bar{n}_{\text{H}}} - \frac{Q}{\bar{t}_{\text{rec}}}
$$
(2.5)

 $rac{dQ}{dt} = \frac{n_{\text{ion}}}{\bar{n}_{\text{H}}} - \frac{Q}{\bar{t}_{\text{rec}}}$ (2.5)
(Madau *et al.* 1999). It is this differential equation—and its equivalent for expanding
helium zones—that statistically describes the transition from a neutral Universe $\frac{dt}{m_H}$ t_{rec}
(Madau *et al.* 1999). It is this differential equation—and its equivalent for expanding
helium zones—that statistically describes the transition from a neutral Universe to
a fully ionized one indepen (Madau *et al.* 1999). It is this differential equation—and its equivalent for expanding helium zones—that statistically describes the transition from a neutral Universe to a fully ionized one independently, for a given U helium zones—that statistically describes the transition from a neutral Universe to
a fully ionized one independently, for a given UV photon emissivity per unit cos-
mological volume \dot{n}_{ion} , of the complex and possib a fully ionized one independently, for a given UV photon emissivity per unit cos-
mological volume \dot{n}_{ion} , of the complex and possibly short-lived emission histories of
individual radiation sources, e.g. on whether t mological volume \dot{n}_{ion} , of the complex and possibly short-lived emission histories of individual radiation sources, e.g. on whether their comoving space density is constant or varies with cosmic time. Initially, whe individual radiation sources, e.g. on whether their comoving space density is constant or varies with cosmic time. Initially, when the filling factor is much less than 1, recombinations can be neglected and the ionized vol stant or varies with cosmic time. Initially, when the filling factor is much less than 1, recombinations can be neglected and the ionized volume increases at a rate fixed solely by the ratio $\dot{n}_{\text{ion}}/\bar{n}_{\text{H}}$. As time 1, recombinations can be neglected and the ionized volume increases at a rate fixed
solely by the ratio $\dot{n}_{\text{ion}}/\bar{n}_{\text{H}}$. As time goes on and more and more Lyc photons are
emitted, radiative recombinations become impo solely by the ratio $\dot{n}_{\text{ion}}/\bar{n}_{\text{H}}$. As time goes on and more and more Lyc photons are
emitted, radiative recombinations become important and slow down the growth of
the ionized volume, until Q reaches unity, the rec emitted, radiative recombinations become important and slow down the growth of
the ionized volume, until Q reaches unity, the recombination term saturates, and
reionization is finally completed (except for the high-dens the ionized volume, until Q reaches unity, the recombination term saturates, and reionization is finally completed (except for the high-density regions far from any source, which are only gradually eaten away (Miralda-E reionization is finally completed (except for the high-density regions far from any source, which are only gradually eaten away (Miralda-Escudé *et al.* 2000)). In the limit of a fast-recombining IGM ($\bar{t}_{rec} \ll t$), one c source, which are only gradually eaten away
limit of a fast-recombining IGM $(\bar{t}_{\text{rec}} \ll t)$,
left-hand side of equation (2.5) and derive

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$$
Q \lesssim \frac{\dot{n}_{\rm ion}}{\bar{n}_{\rm H}} \bar{t}_{\rm rec},\tag{2.6}
$$

 $Q \lesssim \frac{n_{\text{ion}}}{\bar{n}_{\text{H}}} \bar{t}_{\text{rec}},$ (2.6)
i.e. the volume-filling factor of ionized bubbles must be less (or equal) to the number
of Lyc photons emitted per hydrogen atom in one recombination time. In other i.e. the volume-filling factor of ionized bubbles must be less (or equal) to the number
of Lyc photons emitted per hydrogen atom in one recombination time. In other
words because of radiative recombinations, only a fracti i.e. the volume-filling factor of ionized bubbles must be less (or equal) to the number
of Lyc photons emitted per hydrogen atom in one recombination time. In other
words, because of radiative recombinations, only a fract of Lyc photons emitted per hydrogen atom in one recombination time. In other words, because of radiative recombinations, only a fraction $\bar{t}_{\text{rec}}/t \ll 1$ of the photons emitted above 1 ryd is actually used to ionize new words, because of radiative recemitted above 1 ryd is actuall
completely reionized when completely reionized when
 $\dot{n}_{\text{ion}}\bar{t}_{\text{rec}} \gtrsim \bar{n}_{\text{H}},$

i.e. when the emission rate of UV photons exceeds the mean rate of recombinations.

$$
\dot{n}_{\rm ion}\bar{t}_{\rm rec} \gtrsim \bar{n}_{\rm H},\tag{2.7}
$$

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(*c*) *A clumpy Universe*

The simplest way to treat reionization in an inhomogeneous medium is in terms of a clumping factor that increases the effective gas-recombination rate. In this case, The simplest way to treat reionization in an inhomogeneous medium is in terms of a clumping factor that increases the effective gas-recombination rate. In this case, the volume-averaged recombination time is The simplest way to treat reionization in a
of a clumping factor that increases the effective
the volume-averaged recombination time is

the volume-averaged recombination time is
\n
$$
\bar{t}_{\text{rec}} = \left[(1 + 2\chi) \bar{n}_{\text{p}} \alpha_{\text{B}} C \right]^{-1} = 0.06 \text{ Gyr} \left(\frac{\Omega_{\text{B}} h^2}{0.02} \right)^{-1} \left(\frac{1 + z}{10} \right)^{-3} \frac{\bar{n}_{\text{H}}}{\bar{n}_{\text{p}}} C_{10}^{-1}, \qquad (2.8)
$$
\nwhere α_{B} is the recombination coefficient to the excited states of hydrogen (at an assumed gas temperature of 10⁴ K). χ is the helium-to-hydrogen abundance ratio.

where α_B is the recombination coefficient to the excited states of hydrogen (at an assumed gas temperature of 10^4 K), χ is the helium-to-hydrogen abundance ratio, and the factor $C = \langle n^2 \rangle / \bar{n}^2 > 1$ takes into a where α_B is the recombinat
assumed gas temperature of
and the factor $C \equiv \langle n_{\rm p}^2 \rangle / \bar{n}$
photoionized regions (hereaft $_{\rm p}^2 \rangle/\bar{n}_{\rm p}^2$ > ion coefficient to the excited states of hydrogen (at an (10^4 K) , χ is the helium-to-hydrogen abundance ratio, $p^2 > 1$ takes into account the degree of clumpiness of eer $C_{19} \equiv C/10$). If ionized gas with densit assumed gas temperature of 10^4 K), χ is the helium-to-hydrogen abundance ratio,
and the factor $C \equiv \langle n_{\rm p}^2 \rangle / \bar{n}_{\rm p}^2 > 1$ takes into account the degree of clumpiness of
photoionized regions (hereafter $C_{10} \equiv C$ and the factor $C \equiv \langle n_{\rm p}^2 \rangle / \bar{n}_{\rm p}^2 > 1$ takes into account the degree of clumpiness of photoionized regions (hereafter $C_{10} \equiv C/10$). If ionized gas with density $n_{\rm p}$ uniformly filled a fraction $1/C$ of the av photoionized regions (hereafter
filled a fraction $1/C$ of the ava
square density would be $\langle n_{\rm p}^2 \rangle$ =
baryonic mass in photoionized $\binom{2}{p} = n\frac{2}{p}/C$ $p_{10} \equiv C/10$). I
ble volume,
 $p_{\rm p}^2/C = \bar{n}_{\rm p}^2 C$.
s at an overd $2C$ 0). If ionized gas with density n_p uniformly
me, the rest being empty space, the mean
 ${}_{p}^{2}C$. More generally, if f_m is the fraction of
verdensity δ relative to the mean, and the filled a fraction $1/C$ of the available volume, the rest being empty space, the mean
square density would be $\langle n_{\rm p}^2 \rangle = n_{\rm p}^2 / C = \bar{n}_{\rm p}^2 C$. More generally, if $f_{\rm m}$ is the fraction of
baryonic mass in photoioni square density would be $\langle n_{\rm p}^2 \rangle = n_{\rm p}^2 / C = \bar{n}_{\rm p}^2 C$. More generally, if $f_{\rm m}$ is the fraction of baryonic mass in photoionized gas at an overdensity δ relative to the mean, and the remaining (underdense) m baryonic mass in photoionized gas at an
remaining (underdense) medium is distr
occupied by the denser component is s $f_v = f_m / \delta,$ (2.9)

$$
f_{\rm v} = f_{\rm m}/\delta,\tag{2.9}
$$

the density of the diffuse component is

$$
\bar{n}_{\rm p} \frac{1 - f_{\rm m}}{1 - f_{\rm v}},\tag{2.10}
$$

and the recombination rate is larger than that of a homogeneous Universe by a factor of

$$
C = f_{\rm m}\delta + \frac{(1 - f_{\rm m})^2}{1 - f_{\rm v}}\tag{2.11}
$$

 $C = f_m \delta + \frac{\Delta^2 - f_m}{1 - f_v}$ (2.11)
(see, for example, Chiu & Ostriker 2000; Valageas & Silk 1999). It is difficult to
estimate the clumping factor accurately. According to bydrodynamics simulations (see, for example, Chiu & Ostriker 2000; Valageas & Silk 1999). It is difficult to estimate the clumping factor accurately. According to hydrodynamics simulations of structure formation in the IGM (within the framework of (see, for example, Chiu & Ostriker 2000; Valageas & Silk 1999). It is difficult to estimate the clumping factor accurately. According to hydrodynamics simulations of structure formation in the IGM (within the framework of estimate the clumping factor accurately. According to hydrodynamics simulations
of structure formation in the IGM (within the framework of CDM-dominated cos-
mologies), $Ly\alpha$ forest clouds with moderate overdensities, $5 \$ of structure formation in the IGM (within the framework of CDM-dominated cos-
mologies), Ly α forest clouds with moderate overdensities, $5 \le \delta \le 10$, occupy a
fraction of the available volume that is too small for them mologies), Ly α forest clouds with moderate overdensities, $5 \leq \delta \leq 10$, occupy a fraction of the available volume that is too small for them to dominate the clumping at high redshifts (see, for example, Zhang *et al.* fraction of the available volume that is too small for them to dominate the clump-
ing at high redshifts (see, for example, Zhang *et al.* 1998; Theuns *et al.* 1998). In
hierarchical clustering models, it is the virializ ing at high redshifts (see, for example, Zhang *et al.* 1998; Theuns *et al.* 1998). In hierarchical clustering models, it is the virialized gas (with $\delta \approx 180$, if one ignores \geq the slope of the density profile) in hierarchical clustering models, it is the virialized gas (with $\delta \approx 180$, if one ignores
the slope of the density profile) in dark-matter halos with temperatures $\lesssim 10^4$ K
(masses $M \lesssim 10^7 h^{-1} M_{\odot}$) which will p the slope of the density profile) in dark-matter halos with temperatures $\lesssim 10^4$ K (masses $M \lesssim 10^7 h^{-1} M_{\odot}$) which will plausibly boost the recombination rate by large factors as soon as the collapsed mass fract (masses $M \lesssim 10^7 h^{-1} M_{\odot}$) which will plausibly boost the recombination rate by large
factors as soon as the collapsed mass fraction exceeds 0.5%. Halos or halo cores
that are dense and thick enough to be self-shield factors as soon as the collapsed mass fraction exceeds 0.5%. Halos or halo cores
that are dense and thick enough to be self-shielded from UV radiation will stay
neutral and will not contribute to the recombination rate. Th that are dense and thick enough to be self-shielded from UV radiation will stay
neutral and will not contribute to the recombination rate. This is also true of gas
in more massive halos, which will be virialized to higher neutral and will not contribute to the recombination rate. This is also true of gas
in more massive halos, which will be virialized to higher temperatures and ionized
by collisions with thermal electrons. With a large com in more massive halos, whi
by collisions with thermal
of $dn/d \ln M \sim 1000h^3$ M
ca 6h⁻¹ kpc, and to a mas 3 Mpc^{-3} , c will be virialized to higher temperatures and ionized
rons. With a large comoving space density at $z = 9$, corresponding to a mean proper distance of only
ction of 0.04 halos with $T_{\rm c} \approx 10^4$ K will contribute by collisions with thermal electrons. With a large comoving space density at $z = 9$
of $dn/d \ln M \sim 1000h^3 \text{ Mpc}^{-3}$, corresponding to a mean proper distance of only
ca. $6h^{-1}$ kpc, and to a mass fraction of 0.04, halos w of $dn/d \ln M \sim 1000h^3 \text{ Mpc}^{-3}$, corresponding to a mean proper distance of only $ca.6h^{-1}$ kpc, and to a mass fraction of 0.04, halos with $T_v \approx 10^4$ K will contribute significantly, $f_m \delta \approx 7$, to the clumping. Recent cal ca. $6h^{-1}$ kpc, and to a mass fraction of 0.04, halos with $T_v \approx 10^4$ K will contribute significantly, $f_m \delta \approx 7$, to the clumping. Recent calculations by Benson *et al.* (2000), which include instead all halos with T_v significantly, $f_m \delta \approx 7$, to the clumping. Recent calculations by Benson *et al.* (2000), which include instead all halos with $T_v > 10^4$ K and adopt an isothermal density profile with a flat core, give $C \approx 30$ already profile with a flat core, give $C \approx 30$ already at $z = 9$. Because of finite resolution *Phil. Trans. R. Soc. Lond.* A (2000)

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> redshift
Figure 2. (a) Comoving space density of bright QSOs as a function of redshift. The data points
with error bars are taken from different optical and radio surveys (see Madau *et al.* (1999) for Figure 2. (a) Comoving space density of bright QSOs as a function of redshift. The data points
with error bars are taken from different optical and radio surveys (see Madau *et al.* (1999) for
details) (b) Comoving emissio with error bars are taken from different optical and radio surveys (see Madau *et al.* (1999) for details). (b) Comoving emission rate of hydrogen Lyc photons (solid line) from QSOs, compared with the comoving rate of recombinations, $\bar{n}_{\rm H}(0)/\bar{t}_{\rm rec}$ (dashed lines) in an IGM with details). (b) Comoving emission rate of hydrogen Lyc photons (solid line) from QSOs, com-
pared with the comoving rate of recombinations, $\bar{n}_{\rm H}(0)/\bar{t}_{\rm rec}$ (dashed lines) in an IGM with
gas-clumping factor $C = 20, 30,$ pared with the comoving rate of recombinations, $\bar{n}_{\rm H}(0)/t_{\rm rec}$ (dashed lines) in an IGM with
gas-clumping factor $C = 20, 30, 40$). An EdS cosmology with $\Omega_{\rm B} h^2 = 0.02$ and $h = 0.5$ has
been assumed. Models based o been assumed. Models based on photoionization by quasar sources appear to fall short at $z \approx 5$. The data points show the estimated contribution from Lyman-break galaxies at $z \approx 3$ and 4, assuming that the fraction of Lyc photons that escapes the dense HI layers into the galaxy halos and the IGM is 50%. assuming that the fraction of Lyc photons that escapes the dense HI layers into the galaxy halos

> effects, numerical simulations may underestimate clumping: in those of Gnedin $\&$ effects, numerical simulations may underestimate clumping: in those of Gnedin & Ostriker (1997), for example, C rises above unity at $z \lesssim 20$, and grows to $C \sim 10$ (40) at $z \approx 9$ (5) effects, numerical s
Ostriker (1997), for
(40) at $z \approx 9$ (5).
It is important t (40) at $z \approx 9$ (5).
It is important to note that the use of the volume-averaged clumping factor in

(40) at $z \approx 9$ (5).
It is important to note that the use of the volume-averaged clumping factor in
the recombination time-scale is only justified when the size of the HII regions is
much larger than the scale of the clum It is important to note that the use of the volume-averaged clumping factor in
the recombination time-scale is only justified when the size of the HII regions is
much larger than the scale of the clumping, so that the effe the recombination time-scale is only justified when the size of the HII regions is
much larger than the scale of the clumping, so that the effect of many halos within
the ionized volume can be averaged over. This will be a much larger than the scale of the clumping, so that the effect of many halos within
the ionized volume can be averaged over. This will be a good approximation either
at late epochs, when the HII zones have had time to grow the ionized volume can be averaged over. This will be a good approximation either
at late epochs, when the HII zones have had time to grow (or when overlapping
ionized regions from an ensemble of sources are able to proper at late epochs, when the HII zones have had time to grow (or when overlapping
ionized regions from an ensemble of sources are able to properly sample the small-
scale density fluctuations), or at earlier epochs if the ion scale density fluctuations), or at earlier epochs if the ionized bubbles are produced by more luminous sources like quasars or the stars within halos collapsing from high- σ

peaks. As mentioned above, the mean free path between halos having $T_{\rm v} \approx 10^4 \,\rm K$ is $\lambda \sim 6h^{-1}$ kpc at $z = 9$, but their virial radius is only $r_v \approx 0.4h^{-1}$ kpc. It is only on peaks. As mentioned above, the mean free path between halos having $T_v \approx 10^4$ K is $\lambda \sim 6h^{-1}$ kpc at $z = 9$, but their virial radius is only $r_v \approx 0.4h^{-1}$ kpc. It is only on scales greater than $\lambda^3/r_v^2 \approx 2h^{-1}$ Mpc $\lambda \sim 6h^{-1}$ kpc at $z = 9$, but their virial radius is only $r_v \approx 0.4h^{-1}$ kpc. It is or scales greater than $\lambda^3/r_v^2 \approx 2h^{-1}$ Mpc that the clumping can then be averaged and the covering factor of halos within the Strömg and the covering factor of halos within the Strömgren sphere exceeds unity.
 $\,$ 3. Sources of UV photons

(*a*) *Quasars*

(a) *Quasars*
In recent years, several optical surveys (Warren *et al.* 1994; Schmidt *et al.* 1995;
Kennefick *et al.* 1995) have consistently provided evidence for a turnover in the quasi-(a) *Quasars*
In recent years, several optical surveys (Warren *et al.* 1994; Schmidt *et al.* 1995;
Kennefick *et al.* 1995) have consistently provided evidence for a turnover in the quasi-
stellar object (OSO) counts. Th Kennefick *et al.* 1995) have consistently provided evidence for a turnover in the quasi-
stellar object (QSO) counts. The space density of radio-loud quasars also appears Kennefick *et al.* 1995) have consistently provided evidence for a turnover in the quasi-
stellar object (QSO) counts. The space density of radio-loud quasars also appears
to decrease strongly for $z > 3$ (Shaver *et al.* stellar object (QSO) counts. The space density of radio-loud quasars also appears
to decrease strongly for $z > 3$ (Shaver *et al.* 1996), suggesting that the turnover is
indeed real and not an effect on optically selected to decrease strongly for $z > 3$ (Shaver *et al.* 1996), suggesting that the turnover is indeed real and not an effect on optically selected QSOs induced by dust along the line of sight. The density of optically bright and *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 3. Theoretical number-magnitude relation of quasars in the redshift range $4.0 < z < 5.5$. Figure 3. Theoretical number-magnitude relation of quasars in the redshift range $4.0 < z < 5.5$.
The solid line shows the prediction for a 'standard' QSO model, one in which the faint end of
the OSO luminosity function has Figure 3. Theoretical number-magnitude relation of quasars in the redshift range $4.0 < z < 5.5$.
The solid line shows the prediction for a 'standard' QSO model, one in which the faint end of
the QSO luminosity function has The solid line shows the prediction for a 'standard' QSO model, one in which the faint end of
the QSO luminosity function has slope $\beta = 1.64$ and tracks the turnover observed in the space
density of bright quasars at $z \$ the QSO luminosity function has slope $\beta = 1.64$ and tracks the turnover observed in the space
density of bright quasars at $z \ge 3$. The dashed line shows the increased number of sources
expected in the case of a steeper, expected in the case of a steeper, $\beta = 2$, luminosity function and a comoving space density that stays constant above $z = 2.5$. The latter evolution scenario provides, within the errors, enough UV photons to keep the Universe ionized at $z \approx 5$, but appears to be inconsistent with the lack of red, faint stellar objec UV photons to keep the Universe ionized at $z \approx 5$, but appears to be inconsistent with the lack

of red, faint stellar objects observed in the HDF.
has a relatively flat maximum at $1.8 \le z \le 2.8$, and declines gradually at higher
redshifts (figure 2) has a relatively flat
redshifts (figure 2).
The OSO emission s a relatively flat maximum at $1.8 \le z \le 2.8$, and declines gradually at higher dshifts (figure 2).
The QSO emission rate of hydrogen Lyc photons per unit comoving volume, \dot{N}_{Q} , also shown in figure 2. The procedure

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The QSO emission rate of hydrogen Lyc photons per unit comoving volume, \dot{N}_{Q} ,
is also shown in figure 2. The procedure adopted to derive this quantity implies
a large correction for incompletene The QSO emission rate of hydrogen Lyc photons per unit comoving volume, \mathcal{N}_{Q} , is also shown in figure 2. The procedure adopted to derive this quantity implies a large correction for incompleteness at high z. With a is also shown in figure 2. The procedure adopted to derive this quantity implies
a large correction for incompleteness at high z. With a fit to the quasar luminosity
function (LF) that goes as $\phi(L) \propto L^{-\beta}$, with $\beta = 1.$ a large correction for incompleteness at high z. With a fit to the quasar luminosity
function (LF) that goes as $\phi(L) \propto L^{-\beta}$, with $\beta = 1.64$ at the faint end (Pei 1995), the
contribution to the emissivity converges rat function (LF) that goes as $\phi(L) \propto L^{-\beta}$, with $\beta = 1.64$ at the faint end (Pei 1995), the contribution to the emissivity converges rather slowly, as $L^{0.36}$. At $z = 4$, for example, the blue magnitude at the break of contribution to the emissivity converges rather slowly, as $L^{0.36}$. At $z = 4$, for example,
the blue magnitude at the break of the LF is $M_* \approx -25.4$, comparable with or slightly
fainter than the limit of current high-z the blue magnitude at the break of the LF is $M_* \approx -25.4$, comparable with or slightly fainter than the limit of current high-z QSO surveys. While a large fraction, $ca. 90\%$ at $z = 4$ and even higher at earlier epochs, of at $z = 4$ and even higher at earlier epochs, of the ionizing emissivity shown in the figure is, therefore, produced by quasars that have not actually been observed, and are assumed to be present based on an extrapolation figure is, therefore, produced by quasars that have not actually been observed, and are figure is, therefore, produced by quasars that have not actually been observed, and are
assumed to be present based on an extrapolation from lower redshifts, it is also fair
to ask whether an excess of low-luminosity QSOs, assumed to be present based on an extrapolation from lower redshifts, it is also fair
to ask whether an excess of low-luminosity QSOs, relative to the best-fit LF, could
actually boost the estimated Lyc emissivity at early to ask whether an excess of low-luminosity QSOs, relative to the best-fit LF, could
actually boost the estimated Lyc emissivity at early epochs. The interest in models in
which the quasar LF significantly steepens with loo actually boost the estimated Lyc emissivity at early epochs. The interest in models in
which the quasar LF significantly steepens with lookback time—and, therefore, which
predict many more QSOs at faint magnitudes than the which the quasar LF significantly steepens with lookback time—and, therefore, which
predict many more QSOs at faint magnitudes than the extrapolation of Pei's (1995)
fitting functions—stems from recent claims of a strong predict many more QSOs at faint magnitudes than the extrapolation of Pei's (1995)
fitting functions—stems from recent claims of a strong linear correlation between
bulge and observed black-hole masses (Magorrian *et al.* 1 fitting functions—stems from recent claims of a strong linear correlation between
bulge and observed black-hole masses (Magorrian *et al.* 1998), linked to the steep
mass function of dark-matter halos predicted by hierarc bulge and observed black-hole masses (Magorrian *et al.* 1998), linked to the steep mass function of dark-matter halos predicted by hierarchical cosmogonies (see, for example, Haehnelt *et al.* 1998; Haiman & Loeb 1998). A mass function of dark-matter halos predicted by hierarchical cosmogonies (see, for example, Haehnelt *et al.* 1998; Haiman & Loeb 1998). As discussed by Haiman *et al.* (1999), the space density of low-luminosity quasars al. (1999), the space density of low-luminosity quasars at high z is constrained by *Phil. Trans. R. Soc. Lond.* A (2000)

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the observed lack of red, unresolved faint objects in the Hubble deep field (HDF). Down to a 50% completeness limit of $V_{AB} = 29.6$ ($I_{AB} = 28.6$), no $z > 4$ quasar the observed lack of red, unresolved faint objects in the Hubble deep field (HDF).
Down to a 50% completeness limit of $V_{AB} = 29.6$ ($I_{AB} = 28.6$), no $z > 4$ quasar candidates have actually been found by Conti *et al.* (1 Down to a 50% completeness limit of $V_{AB} = 29.6$ ($I_{AB} = 28.6$), no $z > 4$ quasar candidates have actually been found by Conti *et al.* (1999): by contrast, about 10 objects would be predicted by a QSO evolution model cha candidates have actually been found by Conti *et al.* (1999): by contrast, about 10 objects would be predicted by a QSO evolution model characterized by a steep LF with slope $\beta = 2$ and a comoving space density that rema objects would be predicted by a QSO evolution model characterized by a steep LF with slope $\beta = 2$ and a comoving space density that remains constant above $z = 2.5$ instead of dropping (figure 3), and chosen to boost the instead of dropping (figure 3), and chosen to boost the emission rate of UV photons at $z \sim 5$ by a factor of 5. A large population of faint active galactic nuclei at high instead of dropping (figure 3), and chosen to boost the emission rate of UV photons at $z \sim 5$ by a factor of 5. A large population of faint active galactic nuclei at high z would still be consistent with the data if, at at $z \sim 5$ by a factor of 5. A large population of faint active galactic nuclei at high z would still be consistent with the data if, at these faint magnitude levels and high image resolution, the host galaxies of activ z would still be consistent
image resolution, the host
Hubble Space Telescope.

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(*b*) *Star-forming galaxies*

Galaxies with ongoing star formation are another obvious source of Lyc photons. Galaxies with ongoing star formation are another obvious source of Lyc photons.
The recent progress in our understanding of faint galaxy data, made possible by the identification of star-forming galaxies at $2 \le z \le 4$ in Galaxies with ongoing star-formation are another obvious source of Lyc photons.
The recent progress in our understanding of faint galaxy data, made possible by the
identification of star-forming galaxies at $2 \le z \le 4$ in The recent progress in our understanding of faint galaxy data, made possible by the
identification of star-forming galaxies at $2 \le z \le 4$ in ground-based surveys and in the
HDF, has provided new clues to the long-standing identification of star-forming galaxies at $2 \leq z \leq 4$ in ground-based surveys and in the HDF, has provided new clues to the long-standing issue of whether galaxies at high redshifts can provide a significant contributio HDF, has provided new clues to the long-standing issue of whether galaxies at high redshifts can provide a significant contribution to the ionizing background flux. Since the rest-frame UV continuum at 1500 Å (redshifted redshifts can provide a significant contribution to the ionizing background flux. Since
the rest-frame UV continuum at 1500 Å (redshifted into the visible band for a source
at $z \approx 3$) is dominated by the same short-lived the rest-frame UV continuum at 1500 Å (redshifted into the visible band for a source
at $z \approx 3$) is dominated by the same short-lived, massive stars that are responsible
for the emission of photons shortward of the Lyman at $z \approx 3$) is dominated by the same short-lived, massive stars that are responsible
for the emission of photons shortward of the Lyman edge, the needed conversion
factor, about one Lyc photon every 10 photons at 1500 Å, for the emission of photons shortward of the Lyman edge, the needed factor, about one Lyc photon every 10 photons at 1500 Å, is fairly insensi assumed IMF and is independent of the galaxy history for $t \gg 10^{7.3}$ yr.
Com factor, about one Lyc photon every 10 photons at 1500 Å, is fairly insensitive to the assumed IMF and is independent of the galaxy history for $t \gg 10^{7.3}$ yr.
Composite UV luminosity functions of Lyman-break galaxies (L

assumed IMF and is independent of the galaxy history for $t \gg 10^{7.3}$ yr.
Composite UV luminosity functions of Lyman-break galaxies (LBGs) at $z \approx 3$ and $z \approx 4$ have recently been derived by Steidel *et al.* (1999). The Composite UV luminosity functions of Lyman-break galaxies (LBGs) at $z \approx 3$ and $z \approx 4$ have recently been derived by Steidel *et al.* (1999). They are based on a large catalogue of spectroscopically and photometrically s $z \approx 4$ have recently been derived by Steidel *et al.* (1999). They are based on a large catalogue of spectroscopically and photometrically selected galaxies from the ground-based and HDF samples, and span a factor of aro catalogue of spectroscopically and photometrically selected galaxies from the ground-
based and HDF samples, and span a factor of around 40 in luminosity from the faint
to the bright end. Integrating these LFs over all lu to the bright end. Integrating these LFs over all luminosities $L > 0.1L^*$, and using the ground-
om the faint
, and using
nd constant based and HDF samples, and span a factor of around 40 in luminosity from the faint
to the bright end. Integrating these LFs over all luminosities $L > 0.1L^*$, and using
the conversion $L(1500)/L(912) \approx 6$ valid for a Salpe star-formation rate, we derive for the comoving emissivities at 1 ryd the values of the conversion $L(1500)/L(912) \approx 6$ valid for a Salpeter mass function and constant star-formation rate, we derive for the comoving emissivities at 1 ryd the values of $9 \pm 2 \times 10^{25} h$ erg s⁻¹ Hz⁻¹ Mpc⁻³ at $z \approx 3$, star-formation rate, we derive for the comoving emissivities at 1 ryd the values of $9 \pm 2 \times 10^{25} h$ erg s⁻¹ Hz⁻¹ Mpc⁻³ at $z \approx 3$, and $7 \pm 2 \times 10^{25} h$ erg s⁻¹ Hz⁻¹ Mpc⁻³ at $z \approx 4$, about four times higher $9 \pm 2 \times 10^{25} h$ erg s⁻¹ Hz⁻¹ Mpc⁻³ at $z \approx 3$, and $7 \pm 2 \times 10^{25} h$ erg s⁻¹ Hz⁻¹ Mpc⁻³
at $z \approx 4$, about four times higher than the estimated quasar contribution at $z = 3$.
These numbers do not include any at $z \approx 4$, about four times higher than the estimated quasar contribution at $z = 3$.
These numbers do not include any correction for local HI absorption (since the colour excess $E_{912-1500}$ is expected to be small, dus These numbers do not include any correction for local HI absorption (since the colour excess $E_{912-1500}$ is expected to be small, dust extinction can probably be neglected in correcting from observed rest-frame far-UV t excess $E_{912-1500}$ is expected to be small, dust extinction can probably be neglected
in correcting from observed rest-frame far-UV to the Lyman edge). The data points
plotted in figure 2 assume a value of $f_{\text{esc}} = 0.5$ in correcting from observed rest-frame far-UV to the Lyman edge). The data points
plotted in figure 2 assume a value of $f_{\text{esc}} = 0.5$ for the unknown fraction of Lyc
photons that escapes the dense sites of star formation plotted in figure 2 assume a value of $f_{\text{esc}} = 0.5$ for the unknown fraction of Lyc
photons that escapes the dense sites of star formation (not included in our clumping
factor) into the halos and the intergalactic space. photons that escapes the dense sites of star formation (not included in our clumping factor) into the halos and the intergalactic space. Note that, at $z = 3$, LBGs radiate more ionizing photons than QSOs for $f_{\text{esc}} \ge 25$

$98 \text{ for } f_{\text{esc}} \gtrsim 25\%.$
 4. Implications (*a*) *First light*

 (a) First light
We have seen in the previous sections that, in the approximation that the clumping We have seen in the previous sections that, in the approximation that the clumping
can be averaged over, only the photons emitted within one recombination time-scale
can actually be used to jonize new material. As $\bar{t}_{\$ We have seen in the previous sections that, in the approximation that the clumping
can be averaged over, only the photons emitted within one recombination time-scale
can actually be used to ionize new material. As $\bar{t}_{\$ can be averaged over, only the photons emitted within one recombination time-scale
can actually be used to ionize new material. As $\bar{t}_{\text{rec}} \ll t$ at high redshifts, it is possible
to compute, using equation (2.7), a crit can actually be used to ionize new material. As $\bar{t}_{\text{rec}} \ll t$ at high redshifts, it is possible to compute, using equation (2.7), a critical value for the photon emission rate per unit cosmological comoving volume at a cosmological comoving volume at a given epoch, \mathcal{N}_c , independently of the (unknown)
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previous emission history of the Universe: only rates above this value will provide previous emission history of the Universe: only rates above this value will provide
enough UV photons to keep the IGM ionized at that epoch. Expression (2.7) can
then be rewritten as previous emission histo
enough UV photons to
then be rewritten as

$$
\dot{\mathcal{N}}_{c}(z) = \frac{\bar{n}_{H}(0)}{\bar{t}_{rec}(z)} = (10^{51.4} \,\mathrm{s}^{-1} \,\mathrm{Mpc}^{-3}) C_{10} \left(\frac{1+z}{10}\right)^3 \left(\frac{\Omega_{\mathrm{B}}h^2}{0.02}\right)^2. \tag{4.1}
$$

 $N_c(z) = \frac{1}{t_{\text{rec}}(z)} = (10^{24.1} \text{ Npc}^{-1})C_{10}(\sqrt{10}) (\sqrt{0.02})$ (4.1)
The uncertainty on this value is difficult to estimate, as it depends on the clumping
factor and the nucleosynthesis-constrained baryon density. It is int The uncertainty on this value is difficult to estimate, as it depends on the clumping
factor and the nucleosynthesis-constrained baryon density. It is interesting to convert
this rate into a 'minimum' star-formation rate factor and the nucleosynthesis-constrained baryon density. It is interesting to convert $\Omega_{\rm B}h^2 = 0.02$: this rate into a 'minimum' star-formation rate per unit (comoving) volume, $\dot{\rho}_{*}$ (for

$$
\dot{\rho}_{*} = \frac{\dot{\mathcal{N}_{c}} \times 10^{-53.1}}{f_{\rm esc}} \approx (0.12 M_{\odot} \,\text{yr}^{-1} \,\text{Mpc}^{-3}) \bigg(\frac{0.5}{f_{\rm esc}}\bigg) C_{10} \bigg(\frac{1+z}{10}\bigg)^3. \tag{4.2}
$$

 $\rho_* = \frac{f_{\text{esc}}}{f_{\text{esc}}} \approx (0.12M_{\odot} \text{ yr} \text{ Mpc}) \left(\frac{f_{\text{esc}}}{f_{\text{esc}}}\right) \left(\frac{10}{10}\right)$. (4.2)
(The conversion factor can be understood by noting that, for each $1M_{\odot}$ of stars
formed 8% goes into massive stars with $M > 2$ (The conversion factor can be understood by noting that, for each $1M_{\odot}$ of stars formed, 8% goes into massive stars with $M > 20M_{\odot}$ that dominate the Lyc luminosity of a stellar population. At the end of the C-burn (The conversion factor can be understood by noting that, for each $1M_{\odot}$ of stars formed, 8% goes into massive stars with $M > 20M_{\odot}$ that dominate the Lyc luminosity of a stellar population. At the end of the C-burn formed, 8% goes into massive stars with $M > 20 M_{\odot}$ that dominate the Lyc luminosity of a stellar population. At the end of the C-burning phase, roughly half of the initial mass is converted into helium and carbon, with the initial mass is converted into helium and carbon, with a mass fraction released as radiation of 0.007 . About 25% of the energy radiated away goes into Lyc phothe initial mass is converted into helium and carbon, with a mass fraction released
as radiation of 0.007. About 25% of the energy radiated away goes into Lyc pho-
tons of mean energy 20 eV. For each $1M_{\odot}$ of stars fo $0.08\!\times\!0.5\!\times\!0.007\!\times\!0.25\!\times\! M_\odot c^2/2$ 25% of the energy radiated away goes into Lyc photons each $1M_{\odot}$ of stars formed every year, we then expect $\frac{2}{20}$ eV yr $\sim 10^{53}$ photons s⁻¹ to be emitted shortward tons of mear
 $0.08 \times 0.5 \times 0$
of 1 ryd.)
Taken at $708 \times 0.5 \times 0.007 \times 0.25 \times M_{\odot}c^2/20$ eV yr $\sim 10^{53}$ photons s⁻¹ to be emitted shortward
1 ryd.)
Taken at face value, equations (4.1) and (4.2) perhaps have a surprising implica-
on. In an inhomogeneous Universe,

of 1 ryd.)
Taken at face value, equations (4.1) and (4.2) perhaps have a surprising implication. In an inhomogeneous Universe, early reionization at $z \sim 9$ requires an ionizing emissivity that is *comparable with or larger* than that radiated by QSOs at the peak tion. In an inhomogeneous Universe, early reionization at $z \sim 9$ requires an ionizing
emissivity that is *comparable with or larger* than that radiated by QSOs at the peak
of their activity, $z \approx 3$. In a similar manner, emissivity that is *comparable with or larger* than that radiated by QSOs at the peak
of their activity, $z \approx 3$. In a similar manner, photoionization by massive stars can
only play a role if the star-formation density at of their activity, $z \approx 3$. In a similar manner, photoionization by massive stars can
only play a role if the star-formation density at this epoch were significantly larger
than the value directly 'observed' (i.e. uncorre only play a role if the star-formation density at this epoch were significantly larger than the value directly 'observed' (i.e. uncorrected for dust reddening) at $z = 2$ (Madau *et al.* 1998).

(*b*) *Delayed* HeII *reionization*

Because of its higher ionization potential and the steep spectra of UV radiation sources, the most abundant (by a factor of *ca*. 100) absorbing ion in the post-Because of its higher ionization potential and the steep spectra of UV radiation sources, the most abundant (by a factor of ca 100) absorbing ion in the post-
reionization Universe is not HI but HeII. The importance of i tion sources, the most abundant (by a factor of *ca*. 100) absorbing ion in the post-
reionization Universe is not HI but HeII. The importance of intergalactic helium
in the context of this study stems from the possibilit in the context of this study stems from the possibility of detecting the effect of 'incomplete' HeII reionization in the spectra of $z \sim 3$ quasars as, depending on in the context of this study stems from the possibility of detecting the effect of 'incomplete' HeII reionization in the spectra of $z \sim 3$ quasars as, depending on the clumpiness of the IGM (Madau & Meiksin 1994), the ph 'incomplete' HeII reionization in the spectra of $z \sim 3$ quass
the clumpiness of the IGM (Madau & Meiksin 1994), the photonized helium may be delayed until much later than for HI.
Since HI and HeI do not absorb a signific e clumpiness of the IGM (Madau & Meiksin 1994), the photoionization of singly
nized helium may be delayed until much later than for HI.
Since HI and HeI do not absorb a significant fraction of $h\nu > 54.4 \text{ eV}$ photons, t

ionized helium may be delayed until much later than for HI.
Since HI and HeI do not absorb a significant fraction of $h\nu > 54.4 \text{ eV}$ photons, the
problem of HeII reionization can be decoupled from that of other ionizati Since HI and HeI do not absorb a significant fraction of $h\nu > 54.4$ eV problem of HeII reionization can be decoupled from that of other ionization (2.5) for expanding HeIII regions becomes the equivalent of equation (2.5) for expanding HeIII regions becomes

$$
\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\dot{n}_{\text{ion4}}}{\bar{n}_{\text{He}}} - \frac{Q}{\bar{t}_{\text{HeIII}}},\tag{4.3}
$$

where \dot{n}_{ion4} now includes only photons above 4 ryd, and \bar{t}_{HeIII} is 6.5 times shorter than
the hydrogen recombination time-scale if ionized hydrogen and doubly ionized helium where \dot{n}_{ion4} now includes only photons above 4 ryd, and \bar{t}_{HeIII} is 6.5 times shorter than the hydrogen recombination time-scale if ionized hydrogen and doubly ionized helium *Phil. Trans. R. Soc. Lond.* A (2000)

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redshift
Figure 4. The evolution of the HeIII filling factor as a function of redshift in an inhomogeneous
Universe where photoionization is dominated by OSOs turning over at $z \geq 3$. From right to Figure 4. The evolution of the HeIII filling factor as a function of redshift in an inhomogeneous
Universe, where photoionization is dominated by QSOs turning over at $z \gtrsim 3$. From right to
left the three curves assume Universe, where photoionization is dominated by QSOs turning over at $z \ge 3$. From right to left, the three curves assume a constant clumping factor of $C = 10$, 20 and 30. The QSO photon Universe, where photoionization is dominated by QSOs turning over at $z \gtrsim 3$. From right to left, the three curves assume a constant clumping factor of $C = 10$, 20 and 30. The QSO photon spectrum is assumed to vary as left, the three curves assume a constant clumping factor of $C = 10$, 20 and 30 spectrum is assumed to vary as $\nu^{-2.8}$ shortward of the hydrogen Lyman e ionization of HeII is never completed before $z = 3$ in models with

ionization of HeII is never completed before $z = 3$ in models with $C \ge 10$.
have similar clumping factors.[†] It is interesting to note that, if the intrinsic photon
spectrum of ionizing sources has slope $\dot{n}(v) \propto v^{-2.8$ have similar clumping factors.[†] It is interesting to note that, if the intrinsic photon spectrum of ionizing sources has slope $\dot{n}(\nu) \propto \nu^{-2.8}$, the first terms on the right-hand side of equations (2.5) and (4.3) are have similar clumping factors.[†] It is interesting to note that, if the intrinsic photon spectrum of ionizing sources has slope $\dot{n}(\nu) \propto \nu^{-2.8}$, the first terms on the right-hand side of equations (2.5) and (4.3) are spectrum of ionizing sources has slope $\dot{n}(\nu) \propto \nu^{-2.8}$, the first terms on the right-hand
side of equations (2.5) and (4.3) are actually equal, and a significant delay between
the complete overlapping of HII and HeIII side of equations (2.5) and (4.3) are actually equal, and a significant delay between
the complete overlapping of HII and HeIII regions can only arise if recombinations
are important. This effect is illustrated in fig the complete overlapping of HII and HeIII regions can only arise if recombinations plotted for a QSO-photoionization model with a source decline at high redshifts: HeII of the HeIII filling factor (obtained by numerical integration of equation (4.3)) is
plotted for a QSO-photoionization model with a source decline at high redshifts: HeII
reionization is never completed before $z = 3$ in plotted for a QSO-photoionization model with a source decline at high redshifts: HeII
reionization is never completed before $z = 3$ in models with $C \ge 10$. A significant
contribution to the UV background at 4 ryd from ma reionization is never completed before $z = 3$ in models with $C \gtrsim 10$. A significant contribution to the UV background at 4 ryd from massive stars, which could push the helium reionization epoch to higher redshifts, has contribution to the UV background at 4 ryd from massive stars, which could push
the helium reionization epoch to higher redshifts, has been traditionally ruled out on
the grounds that the ratio between the number of HeII the helium reionization epoch to higher redshifts, has been traditionally ruled out on
the grounds that the ratio between the number of HeII and HI Lyc photons emitted
from low-metallicity starbursts is only *ca*. 2% (Leit from low-metallicity starbursts is only $ca.2\%$ (Leitherer & Heckman 1995), five times smaller than in typical QSO spectra. It has recently been pointed out by Tumlinson & Shull (2000), however, that metal-free stars exhi smaller than in typical QSO spectra. It has recently been pointed out by Tumlinson smaller than in typical QSO spectra. It has recently been pointed out by Tumlinson & Shull (2000), however, that metal-free stars exhibit higher effective temperatures and dramatically harder stellar spectra, particularly & Shull (2000), however, that metal-free stars exhibit higher effective temperatures
and dramatically harder stellar spectra, particularly in the HeII continuum. This
enhanced He-ionizing capability of Pop III stars could and dramatically
enhanced He-ioniz
for reionization.
To date variou To date, various studies of the HeII Ly α forest in the spectra of distant QSOs
 \overline{O} To date, various studies of the HeII Ly α forest in the spectra of distant QSOs

(Hogan *et al.* 1997; Reimers *et al.* 1997; Heap *et al.* 2000) have revealed patchy absorption with low HeII opacity 'voids' alternating several-Mpc-sized regions with To date, various studies of the HeII Ly α forest in the spectra of distant QSOs (Hogan *et al.* 1997; Reimers *et al.* 1997; Heap *et al.* 2000) have revealed patchy absorption with low HeII opacity 'voids' alternating

absorption with low HeII opacity 'voids' alternating several-Mpc-sized regions with
† This last assumption appears, however, to be rather dubious: the reason is that self-shielding of HeII
Lyc radiation occurs at much lowe [†] This last assumption appears, however, to be rather dubious: the reason is that self-shielding of HeII Lyc radiation occurs at much lower hydrogen columns than self-shielding of photons at 1 ryd (by about a factor of [†] This last assumption appears, however, to be rather dubious: the reason is that self-shielding of HeII Lyc radiation occurs at much lower hydrogen columns than self-shielding of photons at 1 ryd (by about a factor of Lyc radiation occurs at much lower hydrogen columns than self-shielding of photons at 1 ryd (by about a factor of $\frac{1}{2}S$, where the spectral 'softness', S, is the ratio of the radiation flux at the hydrogen Lyman edge a factor of $\frac{1}{2}S$, where the spectral 'softness', S, is the ratio of the radiation flux edge to the flux at 4 ryd), and self-shielded gas will remain neutral and not ad
rate. Ionized hydrogen may then be more clumpy *Phil. Trans. R. Soc. Lond.* A (2000)

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vanishing flux. These observations suggest that helium absorption does not increase smoothly with lookback time, but rather does so in the abrupt manner expected in vanishing flux. These observations suggest that helium absorption does not increase
smoothly with lookback time, but rather does so in the abrupt manner expected in
the final stages of inhomogeneous reionization by quasar smoothly with lookback time, but rather does so in the abrupt manner expected in
the final stages of inhomogeneous reionization by quasar sources. Radiative transfer
effects during HeII reionization could affect the therm effects during HeII reionization could affect the thermal history of the IGM (Abel $\&$ Haehnelt 1999). Here, it is important to remark that, while delayed HeII reionization in a clumpy Universe appears to be naturally linked to the observed decline in the space density of quasars beyond $z \sim 3$, the complete overlapping of HeIII regions in a clumpy Universe appears to be naturally linked to the observed decline in the space density of quasars beyond $z \sim 3$, the complete overlapping of HeIII regions instead occurs much earlier $(z \gg 5)$ in models that pre space density of quasars beyond $z \sim 3$, then instead occurs much earlier $(z \gg 5)$ in most high redshifts (Haiman & Loeb 1998).

at high redshifts (Haiman & Loeb 1998).
I thank my collaborators, T. Abel, F. Haardt, Z. Haiman and M. Rees, for many useful discus-
sions on the tonics discussed here I thank my collaborators, T. Abel, I
sions on the topics discussed here.

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